STUDY OF BIOMASS PELLET NETWORK MODEL FOR HEAT AND MASS TRANSPORT IN POROUS MEDIA THROUGH GAS CHANNELS

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Abstract. In order to further increase efficiency while reducing harmful emissions, clever control methods have been proposed, e.g. using external electric and magnetic fields. We consider that minimalistic mathematical models are required if heat and mass transfer processes within the granule and the controlling device are low computational power. The authors develop the network model presented in previous publications, further preserving the topology of the network model, consisting of nodes and channels connecting the nodes, with one-dimensional gas dynamics equations governing the gas flow between the nodes. In addition, mass conservation laws are used at the nodes to couple the gas dynamics equations on the various nodes. The resistance to gas flow between the nodes in the two models is described by different parameters – permeability coefficient for the simple network model and channel length and diameter for the alternative model; these can be customized to apply both methods to the main problem. This paper deals with several stages of biomass granule production virtual material design process. At first, a network model describes the biomass granule's internal structure, and heat, mass transfer, and chemical reaction processes are modeled on the network. Finally, we compare the results of the two models for some representative geometries. The results show good agreement if the network model is a sufficiently good discretization of the one-dimensional pipe network model. The results also depend on the geometry of the model.

Keywords: network model, biomass granules, gas channels, hyperbolic heat equation.

Introduction

In recent years, it has become more and more important to introduce an alternative to fossil fuels. Biomass is a sustainable renewable energy source that can help reduce greenhouse gas emissions because it emits fewer greenhouse gases when burned than fossil fuels [1]. Still, it can be more expensive than traditional fossil fuels, primarily if not produced locally but imported. Because of these advantages and challenges, continued research on biomass as a renewable energy source is critical to understanding how best to use this resource while minimizing its negative impacts. Therefore, a promising approach in many applications (including heating houses, transportation biofuel production, and even electricity generation) is biomass granules, composed of wood, straw, peat, etc., and are a common natural resource in Latvia. For this reason, Latvia is an excellent example of increasing biomass consumption in various industries [2]. Recently, both to protect the environment and due to the increase in the price of fossil fuel, more and more people are deciding to heat their homes with granule boilers. However, these processes are complex, resulting from burning biomass granules in a porous medium, which needs to be understood. Thus, a comprehensive mathematical model is required to describe a complex biomass structure**'**s heat and mass transport phenomena.

Consequently, this paper presents a study on a biomass granules network model for heat and mass transport in porous media through gas channels. Our proposed model assumes that the granules form a network pattern that allows the transport of gases and heat through interconnected channels (facets connect vertices). Like network models, there is dynamic modeling of gases on the same gas pipelines as the essential Euler generation [3; 4]. In a porous medium, heat is also transferred by gas flow, but mass transport in a porous medium can be modeled using delay differential equations [5]. At the same time, the hyperbolic heat conduction equation can be considered a delay differential equation [6]. We want to study precisely how delay effects affect the solution, so we compare the heat conduction equation without delay (ordinary differential equation) with the hyperbolic heat conduction equation with delay. In this paper, due to limited space, we consider the equations without heat sources (nonlinearities).

Accurate simulations of numerical methods are given particular importance at pre-treatment of biomass because it is impossible to find standard methods for solving such a task. After all, biomass burning is a complex process, e.g. thermochemical conversion processes [7]. There have been similar studies on biomass, like essential characteristics of microwave pre-treatment can improve energy efficiency [8; 9].

Materials and methods

This work is based on the one-dimensional hyperbolic heat conduction equation given in the form with temperature distribution over time. Obtain the equation for two variables q and u. On the right is the flow and on the left is the pressure [10; 11]:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + f(x,t); q = -k\frac{\partial u}{\partial x}. \quad (1)$$

The function *f* represents a source term that affects the evolution of the function u concerning the time *t* and the spatial variable *x*. The boundary condition

$$q = -k\frac{\partial u}{\partial x}$$

is the heat flow condition specifying the boundary heat flow rate. The thermal diffusion equation is:

$$a^2 = \frac{k}{c\rho},$$

which describes the material's thermal conductivity *k*, specific heat capacity *c* and density ρ with its thermal diffusivity *a*. Expressing

$$\frac{\partial q}{\partial x} = -c\rho\frac{\partial u}{\partial t}$$

and making transformations, we get:

$$\begin{cases} \tau\dfrac{\partial q}{\partial t} + q = -k\dfrac{\partial u}{\partial x} \\ \dfrac{\partial u}{\partial t} = \dfrac{1}{c\rho}\left(-k\dfrac{\partial u}{\partial x}\dfrac{\partial}{\partial x}\right) \end{cases} \quad (2)$$

$$\begin{cases} \dfrac{\partial q}{\partial t} = -\dfrac{1}{\tau}q - \dfrac{k}{\tau}\dfrac{\partial u}{\partial x} \\ \dfrac{\partial u}{\partial t} = -\dfrac{1}{c\rho}\dfrac{\partial q}{\partial x} \end{cases} \quad (3)$$

Writing it down as one equation can be done as follows so that it does not have to be made as a system:

$$\frac{\partial^2 u}{\partial t^2} = -\frac{1}{c\rho}\frac{\partial^2 q}{\partial x \partial t} = -\frac{1}{c\rho}\frac{\partial q}{\partial t}\frac{\partial}{\partial x} = -\frac{1}{c\rho}\left(-\frac{1}{\tau}q - \frac{k}{\tau}\frac{\partial u}{\partial x}\right)\frac{\partial}{\partial x} = \frac{1}{c\rho\tau}\frac{\partial q}{\partial x} + \frac{k}{c\rho\tau}\frac{\partial^2 u}{\partial x^2} =$$

$$-\frac{1}{\tau}\frac{\partial u}{\partial t} + \frac{k}{c\rho\tau}\frac{\partial^2 u}{\partial x^2} \Rightarrow \quad (4)$$

$$\Rightarrow \frac{\partial^2 u}{\partial t^2} + \frac{1}{\tau}\frac{\partial u}{\partial t} = \frac{k}{c\rho\tau}\frac{\partial^2 u}{\partial x^2} \quad (5)$$

When thermal diffusivity accepting the limit as τ approaches zero, it leads to instantaneous heat propagation throughout the material, $\tau \to 0$, then

$$\frac{\partial u}{\partial t} = \frac{k}{c\rho}\frac{\partial^2 u}{\partial x^2}.$$

For our model we use the finite volume discretization method. For simplicity, we can rewrite the hyperbolic heat conduction equation in the following form:

$$\begin{cases} \tau\dfrac{\partial q}{\partial t} + q = -k\dfrac{\partial u}{\partial x} \\ c\rho\dfrac{\partial u}{\partial t} = -\dfrac{\partial q}{\partial x} \end{cases}. \quad (6)$$

With Neumann and Dirichlet boundary conditions:

$$\begin{cases} q|_{x=l} = qd \\ q|_{x=0} = u_s \end{cases}. \tag{7}$$

The boundary conditions at the pipe edge beginning node $x = 0$ is giving by the pressure velocity u_s, but the boundary conditions at the end of the edge node $x = l$ represent mass flow q_d.

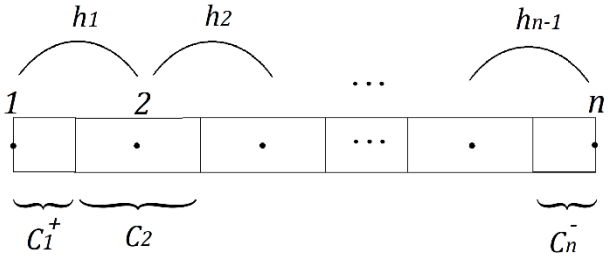

Fig. 1. **Finite volume discretization cell partition**

$$C_2 = \begin{cases} \tau h_2 q_2' + h_2 q_2 = -ku\,|_{x_1}^{x_2} = -k\left(\dfrac{u_3 u_2}{2} - \dfrac{u_2 u_1}{2}\right) = \dfrac{k}{2}(u_3 - u_1) \\ c h_2 u_2' = -\int_{C_2} \dfrac{\partial q}{\partial x} dx = -(q_3 - q_1) \end{cases}, \tag{8}$$

where $\tau h_2 q'_2$ and $c h_2 u'_2$ – equal to time step $(h_1 + h_2)/2$.

$$C_1 = C_1^+ = \begin{cases} \tau \dfrac{h_1}{2}\left(\dfrac{3q_1' + q_2'}{4}\right) + \dfrac{h_1}{2}\left(\dfrac{3q_2 + q_2}{4}\right) = -k\left(\dfrac{u_1 u_2}{2} - u_1\right) = -\dfrac{k}{2}(u_2 - u_1) \\ c \dfrac{h_1}{2}\left(\dfrac{3u_1' + u_2'}{4}\right) = -(u_2 - u_1) \end{cases}, \tag{9}$$

$$C_n = C_n^- = \begin{cases} \tau \dfrac{h_{n-1}}{2}\left(\dfrac{3q_n' + q_{n-1}'}{4}\right) + \dfrac{h_{n-1}}{2}\left(\dfrac{3q_n + q_{n-1}}{4}\right) = -k(u_n - u_{n-1}) \\ c \dfrac{h_{n-1}}{2}\left(\dfrac{3u_n' + u_{n-1}'}{4}\right) = -(u_n - u_{n-1}) \end{cases}, \tag{10}$$

From the set $\{u_1(t), q_1(t), u_n(t), q_n(t)\}$ two quantities are given by boundary conditions; in this case they are creating such a system in matrix form:

$$\begin{bmatrix} M_u & \\ & M_q \end{bmatrix} \begin{bmatrix} u' \\ q' \end{bmatrix} = K \begin{bmatrix} u \\ q \end{bmatrix} = \begin{bmatrix} K_{uu} & K_{uq} \\ K_{qu} & K_{qq} \end{bmatrix} \begin{bmatrix} u \\ q \end{bmatrix}. \tag{11}$$

The matrix K_{uu} is equal to zero, but there are K_{uq}, K_{qu}, K_{qq} diagonal matrices in the following form:

$$M = \begin{bmatrix} M_u & \\ & M_q \end{bmatrix} = h \begin{bmatrix} \frac{3}{8} & \frac{1}{8} & 0 & \cdots & & 0 \\ 0 & 1 & 0 & & & \\ \vdots & & \ddots & & \vdots & \\ & & & 0 & 1 & 0 \\ & 0 & & \cdots & \frac{1}{8} & \frac{3}{8} \end{bmatrix} K_{qu} = \frac{k}{2} \begin{bmatrix} -1 & 1 & 0 & & & & \\ -1 & 0 & 1 & & & & \\ & & & \ddots & & & \\ & & & & -1 & 0 & 1 \\ & & & & 0 & -1 & 1 \end{bmatrix}, \tag{12}$$

$$K_{uq} = \begin{bmatrix} 1 & -1 & 0 & & & & \\ 1 & 0 & -1 & & & & \\ & & & \ddots & & & \\ & & & & 1 & 0 & -1 \\ & & & & 0 & 1 & -1 \end{bmatrix}, \tag{13}$$

$$K_{qq} = -h \begin{bmatrix} \frac{3}{8} & \frac{1}{8} & 0 & \cdots & & & \\ 0 & 1 & 0 & & & & \\ \vdots & & \ddots & & \vdots & & \\ & & & 1 & 0 & 0 & \\ & 0 & & \cdots & \frac{1}{8} & \frac{3}{8} & \end{bmatrix}. \quad (14)$$

The network model is described by a directed graph $G = (E, N)$, where E indicates the edge set, which contains the pipes, but N represents the node set, which joins the edges together. Each node has outgoing and incoming edges.

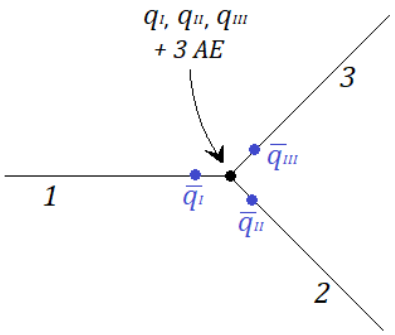

Fig. 2. **Network model node with three edges connected**

At the inner vertex, for each outgoing facet there remains a differential equation against q at the last point, but the differential equation against u is dropped, replacing it with 3 algebraic equations (AE).

$$u_I = u_{II}$$
$$u_{II} = u_{III}$$
$$q_I + q_{II} + q_{III} = 0$$

In the pressure condition case, the pressure at the exiting end of the pipes must be equal to the pressure at the beginning of the entering edges. This is because its edges connect to the same node, ensuring each node has only one pressure value.

In Fig. 3 between images (a) and (b) the connection is that the partial DV is a much better solution in time because many more waves are formed, while the stationary values for both solutions come out to be very similar values. And displaying the solution one node point at a time, we get the following results as Figure 3 shows: (c) the 1st node point is plotted, (d) the 4th node point is plotted, (e) the 8th node point is plotted, and finally, (f) the 11th node point is plotted.

In this experiment, our developed model was compared with the ordinary differential equation on the graph from the previous study [12], based on the equation:

$$\tau \frac{d^2U}{dt^2} + \frac{dU}{dt} = LU \quad (15)$$

where L – Laplace operator.

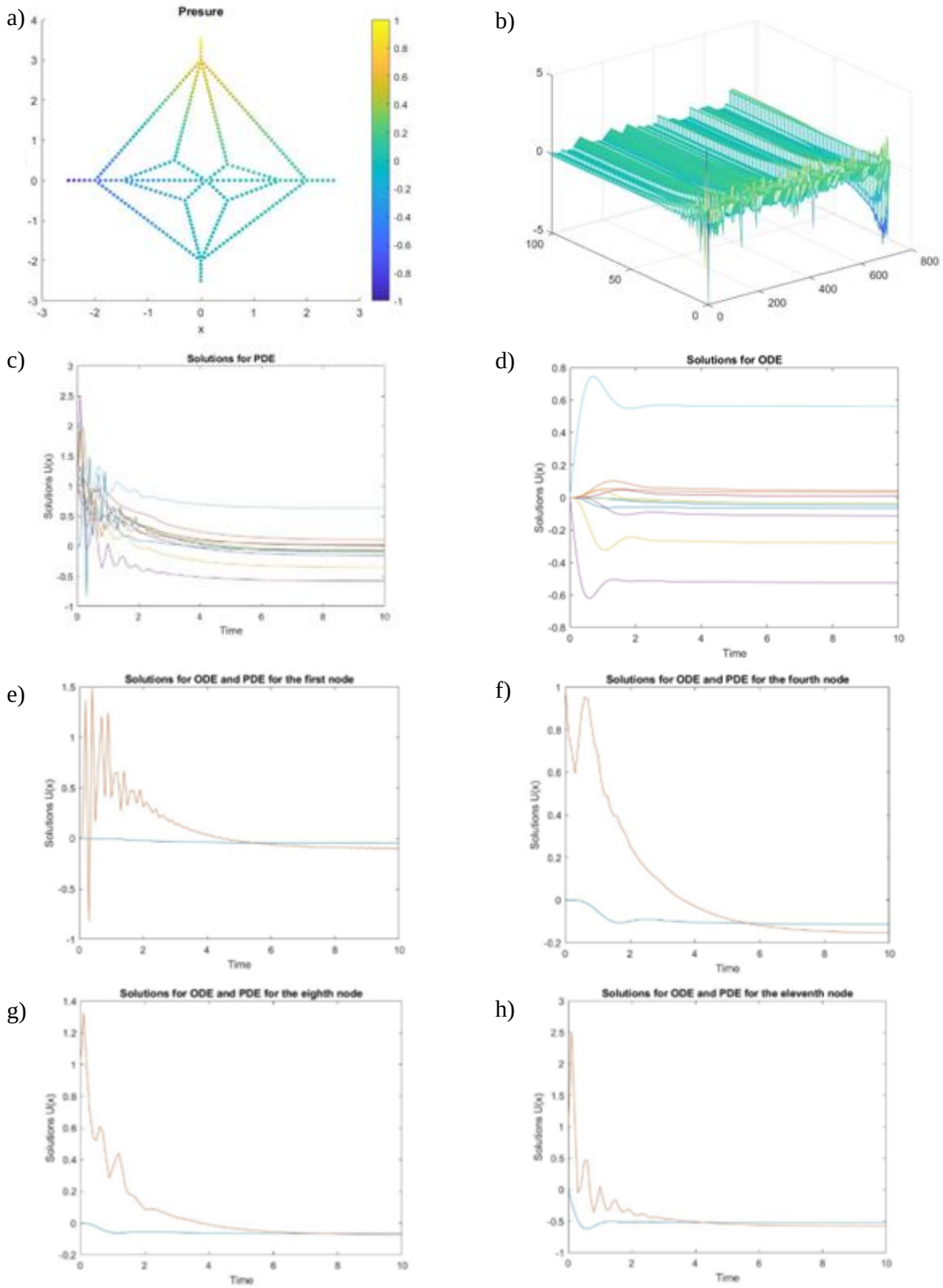

Fig. 3. **Result of the experiment versus the ordinary differential equation:** a – graph with pressure; b – 3D version; c – solutions for PDE; d – solutions for ODE; e-h – ODE and PDE solutions first node, fourth, eighth and eleventh nodes

The same initial conditions were given to both solutions - zero values and non-zero boundary conditions at two external nodes, that is, at the 12th vertex with the value -1 and at the 14th vertex with the value 1, at the 14th and 15th vertices there are zeros. Since the parameter τ is vital because it represents the relaxation time, we have chosen it with a value of 0.3, allowing the fluctuations to decay

quite quickly for both pressure and flow. Also, these tasks have a uniform time grid with consistent time steps.

Conclusions

This paper promotes developing a comprehensive model for combustion of biomass granules in a porous medium through gas channels, which can be helpful in the design and optimization of biomass energy systems. One promising approach to biomass energy is still biomass granules, which can be easily stored and transported, making the granules even more environmentally friendly. Furthermore, the method of partial differential equations is much more accurate, which can also be seen from the resulting graphs. However, it can be observed that the pressure propagates slowly along the graph, so it is essential that the exact solutions of the partial differential equations are used and that the ordinary differential equations on the graph still give some errors.

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